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# Approach to exponential and logarithmic functions through didactic analysis

## Aproximación a las funciones exponencial y logarítmica a través del análisis didáctico

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**ABSTRACT**

**Keywords:**
exponential function, logarithmic function, didactic analysis.

The article presented focuses on the planning and organization of teaching and learning of exponential and logarithmic functions and is born from the need to improve learning in Mathematics. The theoretical framework used is Didactic Analysis, in its four sub-analyses (content, cognitive, instructional and performance) through the design and implementation of a didactic unit and, as a methodology, a mixed-cut study with two groups of fourth-year students from an Argentine educational institution. The didactic unit consists of title, objectives, content, methodology, evaluation and tasks. The results show that the organization of teaching under this approach determines a higher academic performance of students in Mathematics in the subject of exponential and logarithmic functions, since the scores, analyzed by the U Man Whitney test, of the experimental group were higher than those of the control group with a statistical significance of less than 0.05. Likewise, interviews were conducted with the students in the experimental group at the end of the teaching unit, on each of the exercises and problems in it, and their opinions indicated that they reached a more complete understanding of the subject, so that through this intervention they not only improved their grades, but also their understanding of the subject in question.

**RESUMEN**

**Palabras clave:**
función exponencial, función logarítmica, análisis didáctico.

El artículo que se presenta centra su atención en la planificación y organización de la enseñanza y del aprendizaje de la función exponencial y logarítmica y nace de la necesidad de mejora de los aprendizajes en Matemáticas. Se utiliza como marco teórico el Análisis Didáctico, en sus cuatro subanálisis (de contenido, cognitivo, de instrucción y de actuación) mediante el diseño y la implementación de una unidad didáctica y, como metodología, un estudio de corte mixto con dos grupos de estudiantes de cuarto año de una institución educativa argentina. La unidad didáctica se compone de título, objetivos, contenidos, metodología, evaluación y las tareas. Los resultados muestran que la organización de la enseñanza bajo este enfoque determina un mayor rendimiento académico de los alumnos en Matemáticas en el tema de funciones exponenciales y logarítmicas, ya que las puntuaciones, analizadas mediante la prueba U Man Whitney, del grupo experimental fueron superiores al del grupo control con una significancia estadística menor a 0,05. Asimismo, se realizaron entrevistas a los estudiantes del grupo experimental, al finalizar la unidad didáctica, sobre cada uno de los ejercicios y problemas de ésta y sus opiniones denotaron que llegaron a una comprensión más acabada del tema, por lo que mediante esta intervención no solo mejoraron sus calificaciones, sino que además mejoró su comprensión del tema en cuestión.

## Introduction

This research is intended as a contribution to the improvement of Mathematics teaching and learning processes, which is necessary due to the negative results of students' performance in Mathematics in the last decades, particularly in the Exponential and Logarithmic Function, in terms of posing and solving exponential and logarithmic equations and the complete study of these functions, including their graphs, at the middle level.

This work is based on the planning, design and implementation of a didactic unit (See Annex 1) corresponding to the 4th year program of the intermediate level of Mathematics and the topic of Exponential and Logarithmic Functions based on the Didactic Analysis method, with students of a private school with a bachelor's degree in computer science, in the Autonomous City of Buenos Aires, Argentina.

The objective of this article is "To apply Didactic Analysis in a didactic unit of the exponential and logarithmic function in mathematics in the 4th year of secondary school".

The didactic unit is composed of TITLE, OBJECTIVES, CONTENTS, METHODOLOGY, EVALUATION and TASKS.

The implementation of a didactic unit for the teaching and learning of the exponential and logarithmic function, from the Didactic Analysis aims to improve the effective learning that is intended to be achieved, in this context the theoretical framework provided by the Didactic Analysis is reviewed:

According to Martínez, Triviño and Bonilla (2023) Didactic Analysis develops the competence of curricular planning that allows the mathematics teacher to design, develop and evaluate the mathematical content to be managed in the classroom, where the elements, times and ideal conditions for the management process of a didactic unit can be foreseen. (p.43)

Rico, Lupianez and Molina (2013) state that Didactic Analysis consists of four analyses: content, cognitive, instructional and performance.

According to these authors, in the content analysis located in the cultural and conceptual dimension of the curriculum, the teacher must identify, select and organize the meanings of the concepts and procedures of the mathematical topic in question that he/she believes are relevant to plan as approved school content for instruction. To review and organize the concepts and procedures, the way in which they can be represented and the organization of the phenomena and problems to which they can provide solutions, will delimit the curriculum organizers that make up the content analysis.

According to Martínez (2020) who follows Lupiáñez (2009), he differentiates different levels of content analysis: the knowledge that is imparted and that was considered in the course of history, the contents that education laws indicate should be taught, the contents that are proposed for a subject, and the content of a particular topic.

In this stage of the research, the concepts to be taught were identified, based on the corresponding curricular design, linked to the teaching and learning of the exponential and logarithmic function.

Rico et al., (2013) expand with respect to curriculum organizers:

> The content analysis is organized around 3 curriculum organizers:
>
> Representation systems, where the different ways in which the content and its relationships with other concepts and procedures can be represented are considered.
>
> Phenomenology, which considers the phenomena (contexts, situations and problems) that can give meaning to the content under consideration.

> The conceptual structure, which considers the relationships of the concepts and procedures involved in the content studied, taking into account both the mathematical structure of which they are part, as well as the structure of such concepts and procedures. (Rico et al., 2013, p.85)

In the elaboration of the didactic unit, it was considered that the students work with different representation systems:

Algebraic: it is based on graphemes and has its own writing rules.

Verbal: the way in which mathematical entities, their relationships and priorities are expressed verbally; there is specific terminology to differentiate them.

Graph: based on graphs, it is the one that appears when representing relations or functions on Cartesian axes, numbers on the number line, etc.

Tabular: it deals with information through tables.

Specifically, in the tasks "Juggling balls", "Christmas wreaths" and "Richter scale" they worked with algebraic, verbal and graphic representations, in the task "The million" with algebraic, verbal and tabular, and in the task "Fixed term" they used all four types of representations.

In terms of phenomenology, all the problems in the didactic unit were posed with everyday life situations ("Juggling balls", "Christmas wreaths", "Fixed term", "The million" and "Richter scale", the latter being a scale used to measure the intensity of earthquakes)

The conceptual structure, as defined above, was established after a thorough review of the 4th grade Mathematics textbooks. Year: Funciones 2 Altman, S., Comparatore, C and Kurzrok, L. (2008) and Matemática II Molina, A., Féliz O., Laurens, R., Toribio, C. Cueto, R., Michel, D., Carbonell, L., Larcier, N., Lugo, J. and Montes de Oca, R. (2008).

Continuing with the cognitive analysis:

Rico et al., (2013) illustrate in relation to cognitive analysis:

> Cognitive analysis, located in the cognitive dimension of the curriculum, addresses the problem of how schoolchildren learn this mathematical subject. From a constructivist approach (Coll, 2002), the teacher, based on the information obtained in the previous content analysis and the knowledge about school mathematics and its learning, states and organizes learning expectations about this mathematical topic. It also analyzes those limitations that may interfere with learning, and organizes the selection of tasks that will provide schoolchildren with the opportunity to learn. (Rico et al.,2013, p.83)

According to Martínez (2020) who refers to (Rico and Fernández-Cano, (2013), the cognitive analysis, from the curricular approach, takes into account the complexity of the tasks according to the level of depth of the subject in question, the variety of learning objectives they encompass and the obstacles whose domination they entail.

In this work, in the cognitive analysis stage, the learning objectives were determined from the information obtained in the content analysis and the possible limitations that could appear in the learning of this topic were also analyzed from the errors of the diagnostic tasks (see Annex 2) of the previous knowledge. Subsequently, the relevant tasks were determined.

> In instructional analysis, the teacher selects, designs, and sequences the tasks to be used in instruction to achieve the learning expectations he or she has previously specified. It also analyzes the different materials and resources that can be used in their classes and, among other aspects, defines

> the sequencing of tasks and sessions and delimits central aspects of classroom management. (Rico, et al., 2013, p.83)

Martínez (2020) considers that the instructional analysis, after the previous analyses, focuses on the teaching of the mathematical subject in question, and from the curricular approach it is centered on planning the teaching to find as a result the design of the didactic unit, a design that is justified in the content and cognitive analyses. The author then refers to Gómez (2002) who indicates that in the Didactic Analysis procedure there should be a dialogical relationship between all the sub-analyses, each sub-analysis of the Didactic Analysis not only responds to and builds on the previous one, but also, what is needed in one phase can review and modify the analyses carried out in the previous phases.

In this stage, the tasks and their sequence were designed in relation to the objectives set and the number of classes that make up the didactic unit.

Finally, in the performance analysis:

Rico et al., (2013), on performance analysis state:

> The last analysis, the performance analysis, is carried out after implementing the didactic unit and serves the teacher to gather information about: the extent to which the established learning expectations have been achieved, the functionality of the tasks used or the goodness of the evaluation tools put into play. This information is useful in view of the next implementation of the designed unit or at the beginning of the planning of the next topic. (p.83)

Martínez (2020) follows Lupiáñez (2013) who indicates that, after the instructional analysis, the teacher will reflect on the level of adequacy of his teaching-learning proposal by analyzing the results he found. Martínez (2020) adds that performance analysis has two focuses, one consists of reflecting on whether the learning objectives were achieved and whether the students overcame the expected obstacles, in this focus the previous cognitive analysis is essential; and the other focus should be on teaching as a process, since the results obtained by the students are the product of the teacher's actions and the design of the didactic unit, in this last focus the analysis of content and instruction, both previously performed, is essential. Martínez (2020) referring to Lupiáñez (2013) enunciates that at this stage the teacher may consider:

- Weigh whether the instruction was consistent and coherent in selecting and organizing the tasks and content and whether they were conducive to the learning expectations set.
- Establish the level of achievement of learning expectations and the development of specific mathematical competencies achieved by students.
- Verify that difficulties and errors have been overcome.
- Reflect on the timeliness of the teaching resources and materials used.
- Assess the suitability of evaluation instruments to inform and drive learning.

In previous terms, the analysis of results is materialized by establishing the strengths and weaknesses of what has been proposed in order to seek improvements in the following cycles. Therefore, in this analysis, a new design of the didactic unit is born.

The students' learning was evaluated, through their academic performance, after the implementation of the field work of this research.

The development of each of the classes that make up the implementation of the didactic unit consisted of an introductory explanation by the teacher, then the students solved the problems and exercises and developed the tasks to finally deliver the resolutions of the problems and exercises, and in the following class they received the corresponding feedback from the teacher.

# Method

The research methodology applied was a case study with a mixed and interpretative approach. This methodology was applied to two groups, the control group (one of 19 students cohort 2021 with traditional teaching) and the experimental group of 21 students cohort 2022 (with the implementation of the didactic unit of this research), respectively, of 4th year students of the Colegio de Nuestra Señora (Autonomous City of Buenos Aires - Argentine Republic), in the subject Mathematics.

This work involved the design and implementation of a didactic unit for 4 weeks of 4 teaching hours per week, corresponding to the topic Exponential and logarithmic function based on Didactic Analysis in its four phases (Content, Cognitive, Instructional and Performance Analysis), the students had previously studied real numbers, sequences, probability and statistics and cubic functions. As a limitation, the possible lack of training of teachers involved in the Didactic Analysis current is considered a limitation.

According to the results obtained after performing the diagnostic tasks, the topics in which the students presented greater difficulties in relation to their previous knowledge were: common factor, clearing unknowns, fractional exponent, graphing functions, classifying functions, study of functions, asymptotes, inverse function, systems of equations, quadratic function, canceling property, correct use of the calculator and the calculation of the solutions of an equation and the highest percentages of errors were found in the topic of functions, which leads to the fact that later they cannot correctly assimilate the topic of Exponential and Logarithmic Function.

The theoretical framework explored Didactic Analysis, in its four phases (Content, Cognitive, Instructional and Performance Analysis), which frames this research, as described in the introduction.

The academic performance of the students was measured through the grades obtained in the same written exam, and was compared with respect to the learning level of the students of the previous year. It should be clarified that the traditional methodology was applied to the students of the control group, that is, without the application of the Didactic Analysis and without the implementation of the didactic unit based on it, and the methodology described in this article was applied to the students of the current year at that time, who are the experimental group.

Also, after carrying out the corresponding tasks, interviews (see Appendix 4) were conducted with the students in the experimental group in three stages: initial, developmental and final, the results of which appear in the results section of this article.

Finally, academic performance was compared, according to the results obtained in the control and experimental groups, through the Mann Whitney U test in Microsoft Excel. The Mann Whitney U is used for two independent groups, in samples of less than thirty units of analysis, for an ordinal variable in a non-normal distribution, hence the reason for its choice.

# Results

As the students carried out the didactic unit at the end of the tasks, they were interviewed, the results of which show the benefits they perceived in their learning (See Annex 4).

The interviews were conducted with a total of 21 4th grade students. Year.

Grades range from 0 to 10, both inclusive, and the passing grade is equal to or higher than 6, so that those who obtain a grade lower than 6 will fail.

The results obtained in the final evaluation, which served to assess student learning through their academic performance, were as follows:

**TABLE 1**

*Results obtained in the evaluations of both groups*

| Participant number | Control group notes | Experimental group notes |
|---|---|---|
| 1 | 8 | 10 |
| 2 | 3 | 10 |
| 3 | 8 | 7 |
| 4 | 10 | 10 |
| 5 | 5 | 8 |
| 6 | 8 | 10 |
| 7 | 5 | 6 |
| 8 | 8 | 7 |
| 9 | 6 | 6 |
| 10 | 3 | 8 |
| 11 | 8 | 10 |
| 12 | 3 | 8 |
| 13 | 6 | 5 |
| 14 | 5 | 9 |
| 15 | 2 | 8 |
| 16 | 8 | 4 |
| 17 | 8 | 10 |
| 18 | 5 | 9 |
| 19 | 2 | 8 |
| 20 | | 7 |
| 21 | | 9 |

Note. Control group: students cohort 2021 (with traditional teaching) and Experimental group: students cohort 2022 (with the implementation of the didactic unit of this research).

Table 1 shows that in the experimental group the overall scores were higher.

Figure 1 shows the number of passes and the percentage of passes in each group.

**FIGURE 1**

*Number of passes (total number of passes over the total of the corresponding group) and percentage of passes per group*

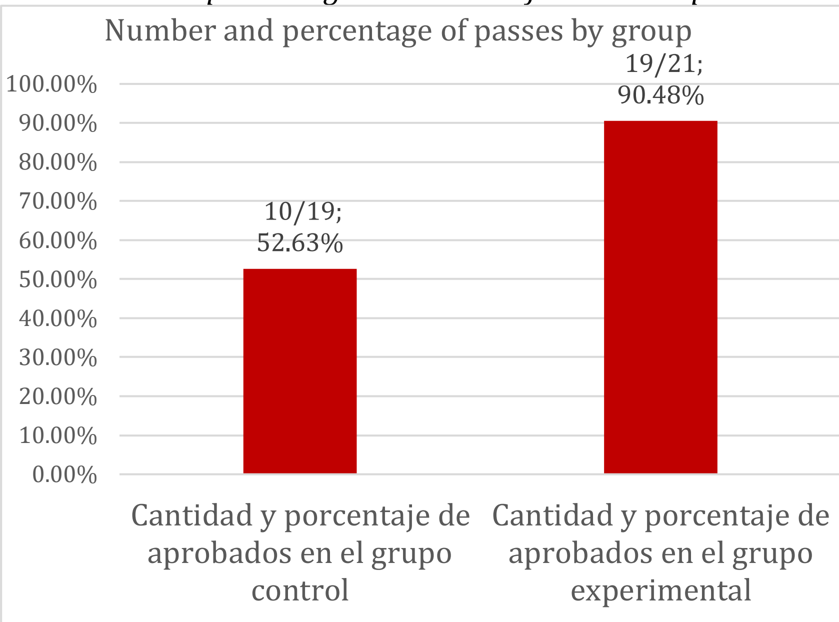



Figure 1 shows how the experimental group has increased the number and, therefore, the percentage of passing grades.

Figure 2 shows the comparison of grade frequencies in both groups, for which the data corresponding to the grades have been divided into 5 intervals, being the values of the grades from 0 to 10 and from 6 the value corresponding to the passing grade:

**FIGURE 2**
*Comparison of grade frequencies in both groups*

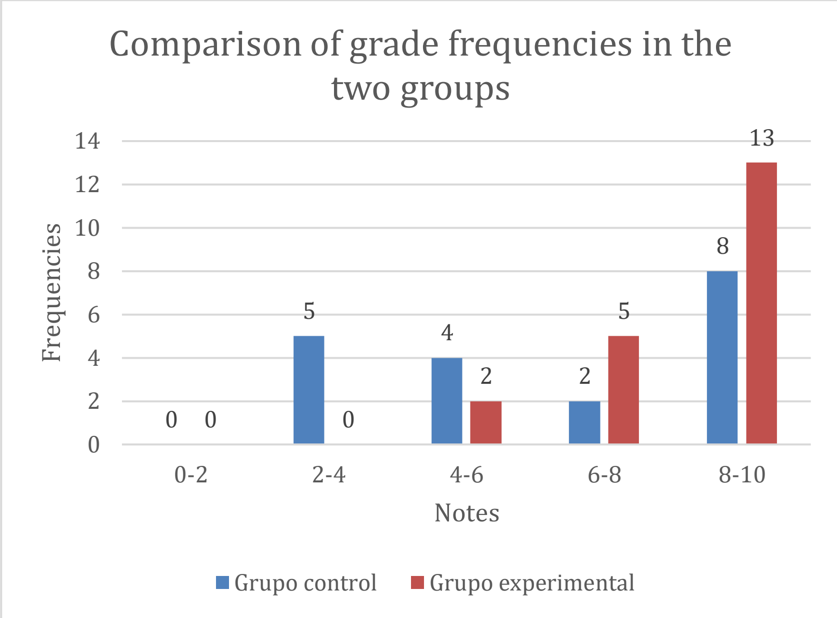



Figure 2 shows that in the experimental group, not only was the number of passers higher, but also, in general, the grades of the failers in the second group were higher than those of the passers.

Table 2 shows the results when applying, for statistical purposes, the Mann Whitney U test used to compare academic performance in both groups:

**TABLE 2**

| Calculation to determine the U Mann Whitney U Result | Amostra 1 | Amostra 2 |
|---|---|---|
| Sample size | 19 | 21 |
| Soma dos Postos (Ri) | 286,5 | 533,5 |
| Median = | 6,00 | 8,00 |
| U = | 96,50 | |
| p-value (one-sided) = | ,0026 | |
| p-value (bilateral) = | ,0053 | |

Note. Results obtained with Bioestat 5.3 statistical software

Table 2 shows that the groups showed statistically significant differences in academic performance where the scores of the control group (median=6) were lower than that of the experimental group (median=8) U=96.50, p=0.0026. A p-value of less than 0.05 is evidence of a statistically significant difference between the two groups.

It is evident that academic performance improves with the application of the Didactic Analysis, since in Figure 1 it can be observed that in the control group 52.63% of the students passed and in the experimental group 90.48% of the students passed, which represents a difference of 37.85% more students passed in the experimental group. at the same time, the improvement is still evident since, as can be seen in Figure 2, in the experimental group not only was the number of passed students higher, but also the number of failed students in the experimental group was higher than in the passed students, in both groups there were no failed students with grades between 0 and 2, with grades between 2 and 4 there were 5 failures in the control group and no failures in the experimental group, with grades between 4 and 6 there were 4 failures in the control group and only 2 in the experimental group, with grades between 6 and 8 there were 5 passes in the experimental group and only 2 passes in the control group and finally with grades between 8 and 10 there were 13 passes in the experimental group and only 8 passes in the control group.

Finally, once the results of the test taken by both groups were obtained, it was demonstrated that academic performance in the subject improved with the application of this didactic proposal.

## Discussion and Conclusions

After implementing the didactic unit, evaluating the students' performance through the written test and conducting interviews in three stages (initial, developmental and final), the following conclusions were reached:

- It was possible to observe an improvement in the academic performance of the experimental group with respect to the control group.
- The results of the interviews that were applied to the students on each of the exercises on which they were asked for their opinion show that they reached a more complete understanding of the subject.
- The Man Whitney U test shows the significant improvement in the academic performance of the experimental group with respect to the control group.

## References


Altman, S., and Comparatore, C., (2008). Matemática. Funciones 2. Ciudad Autónoma de Buenos Aires: Longseller .

Coll, C. (2002) Constructivismo y educación: la concepción constructivista de la enseñanza y el aprendizaje. In Coll, César; Palacios Jesús y Marchessi, Álvaro Desarrollo Psicológico y Educación.Tomo II. Madrid. Spain: Alianza Editorial

Martinez, E., Triviño, J. A. and Bonilla, B. E. (Ed.). (2023). Las prácticas del profesor de matemáticas en formación inicial al diseñar tareas asociadas a la fracción. Florencia, Colombia: Editorial Universidad de la Amazonia

Martinez, S. (2020) diseño, implementación y análisis de una propuesta didáctica para la proporcionalidad en el primer ciclo de secundaria. Valladolid, Spain: Escuela de Doctorado Universidad de Valladolid.

Molina, A., Féliz O., Laurens, R., Toribio, C. Cueto, R., Michel, D., Carbonell, L., Larcier, N., Lugo, J. and Montes de Oca, R. (2008). Matemática II. Ciudad Autónoma de Buenos Aires: Santillana.

Rico, L., Lupianez J. L. and Molina, M. (2013). Análisis Didáctico en Educación Matemática. a. Metodología de investigación, formación de profesores e innovación curricular. Grenada. Spain: Editorial Comares, S.L.

# Annexes

***Annex 1***

*Didactic unit*

*Title:*

Exponential and logarithmic functions

Objectives:

Strengthen the concept of function and its related concepts.

Understand the characteristics, graphical behavior and growth of exponential functions, including the concept of inverse function.

Understand the concept of logarithm, properties of logarithms, characteristics of the logarithmic function, graphical behavior and growth, including the concept of inverse function

Solve exponential and logarithmic equations, optimizing these processes.

Apply the knowledge acquired in the resolution of problems, optimizing these processes

*Contents:*

Review of: density of a substance, area and volume of a sphere, area and perimeter of the circular crown, deduction of formulas, definition of function, graph of functions, quadratic and cubic function, classification of functions, domain, image, zeros, intervals of positivity and negativity, intervals of growth, decrease and constancy, asymptotes and inverse function.

Exponential function, logarithm, properties of logarithms, logarithmic function, logarithmic function, exponential and logarithmic equations. Exercises and problems.

*Methodology:*

First, the problem statement was read several times and the teacher clarified the doubts that arose, then the students solved them and handed in the resolutions to the teacher, and in the following class they received the pertinent corrections and feedback from the teacher.

*Evaluation:*

The evaluation was carried out by means of a written test.

*Tasks:*

Task “Juggling balls”

In a circus they want to build stuffed balls for juggling, calculate:

1. How much padding and how much lining fabric should be purchased per ball if you want to make them 60 mm in diameter filled with rice (density=0.9 g/cm$^3$).
2. Graph the function of the volume of the ball as a function of radius and the area of the ball as a function of radius.
   a. Graph it
   b. Classify it. Justify.
   c. Indicate its domain, image, zeros, intervals of positivity and negativity, intervals of growth, decrease and constancy. Justify.
   d. Find the asymptotes and the inverse function, if any. Justify.

Task “Christmas wreaths”

You want to make cardboard Christmas wreaths for doors, calculate:

1. To deduce the formula of the circular crown, from the formula of the area of the circle.

2. If the crowns are to be made with a diameter of 25 cm and a thickness of 5 cm, calculate how many $m^2$ of cardboard would have to be purchased to make 50 crowns.
3. Find the formula for the perimeter of a circular wreath from the formula for the length of a circumference and calculate the m of Christmas ribbon to be purchased to encircle the outside and inside of a Christmas wreath.
4. Plot the function of crown area as a function of thickness.
    a. Graph it.
    b. Classify it. Justify.
    c. Indicate its domain, image, zeros, intervals of positivity and negativity, intervals of growth, decrease and constancy. Justify.
    d. Find the asymptotes and the inverse function, if any. Justify.

*Exponential function*

Task "Fixed term"

Difference between simple and compound interest:

When an investor places money in a bank for a fixed term that generates for example monthly interest and the investor withdraws the interest he earns each month, this is called SIMPLE INTEREST, in the case that he does not withdraw it and lets it be added to his capital each month to produce new interest, this is called COMPOUND INTEREST.

We place for 4 quarters $100,000 in a bank, which pays 10% interest quarterly, complete the following tables assuming that it does so at simple interest in the first and compound interest in the second, complete, justifying, the missing rows:

| PERIOD | CAPITAL AT THE BEGINNING OF THE PERIOD | INTEREST EARNED IN THE PERIOD | AMOUNT |
|---|---|---|---|
| 1 | $100000 | $10000 | $110000 |
| 2 | $100000 | $10000 | $110000 |
| 3 | $100000 | $10000 | $110000 |
| 4 | $100000 | $10000 | $110000 |

| PERIOD | CAPITAL AT THE BEGINNING OF THE PERIOD | INTEREST EARNED IN THE PERIOD | AMOUNT |
|---|---|---|---|
| 1 | $100000 | $10000 | $110000 |
| 2 | $110000 | $11000 | $121000 |
| 3 | $121000 | $12100 | $132100 |
| 4 | $132100 | $13210 | $145310 |

Now we are going to derive a formula that allows us to calculate the amount of a capital at compound interest after n periods:

| PERIOD | CAPITAL AT THE BEGINNING OF THE PERIOD | INTEREST EARNED IN THE PERIOD | AMOUNT |
|---|---|---|---|

| | | | |
|---|---|---|---|
| 1 | C | I=C. i. 1 (simple interest formula) | $C_1$=C+I $C_1$= C+(C.i.1) = $C_1$=C(1+i) |
| 2 | $C_1$ | I=$C_1$. I. 1 | $C_2$=$C_1$ + I=$C_1$+ $C_1$. I= $C_2$=$C_1$ (1+i) =C (1+i) (1+i) $C_2$= $C(1+i)^2$ |
| 3 | $C_2$ | I=$C_2$. I. 1 | $C_3$=$C_2$ + I=$C_2$+ $C_2$. I= $C_3$=$C_2$ (1+i) =C $(1+i)^2$ (1+i) $C_3$= $C(1+i)^3$ |
| 4 | | | |
| | | | |
| n-1 | | | |
| n | | | |

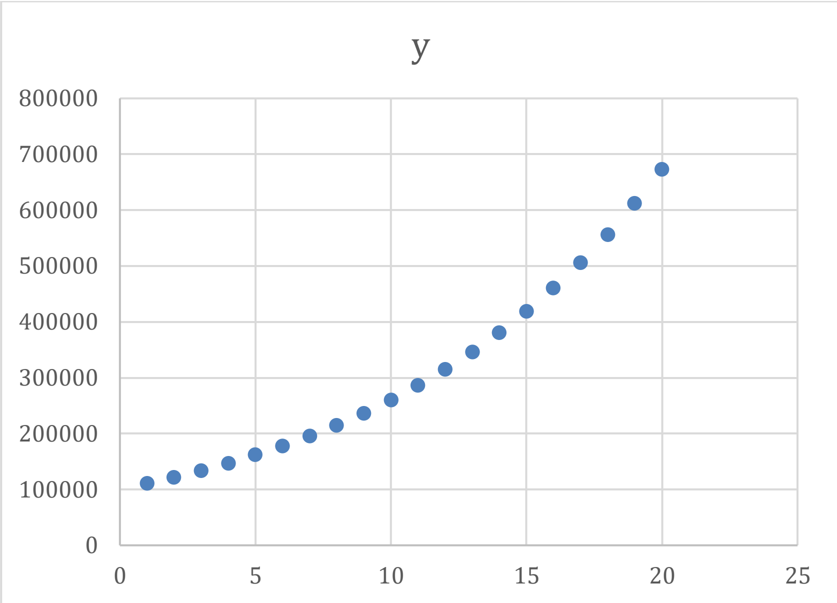
*Activity 1:*

We will now plot the compound interest amount as a function of the period, using a spreadsheet and the formula just derived for the compound interest amount.

The results to be obtained are shown below:

| x | y |
|---|---|
| 1 | 110000 |
| 2 | 121000 |
| 3 | 133100 |
| 4 | 146410 |
| 5 | 161051 |
| 6 | 177156,1 |
| 7 | 194871,71 |
| 8 | 214358,881 |
| 9 | 235794,769 |
| 10 | 259374,246 |
| 11 | 285311,671 |
| 12 | 313842,838 |
| 13 | 345227,121 |
| 14 | 379749,834 |
| 15 | 417724,817 |
| 16 | 459497,299 |
| 17 | 505447,028 |
| 18 | 555991,731 |

| | |
|---|---|
| 19 | 611590,904 |
| 20 | 672749,995 |




Looking at the graph, answer the following question:

- Does the obtained graph represent a function, if the answer is yes:
  a. Classify it
  b. Indicate its domain, image, zeros, intervals of positivity and negativity, intervals of growth, decrease and constancy.
  c. Find the asymptotes and the inverse function, if any.

*Logarithm*

Definition: The logarithm is the exponent to which a given base (positive and not equal to 1) must be raised to obtain the given power.

Examples:

$Log_2 8 = 3$ so $2^3=8$

$Log_3 9=2$ so $3^2=9$

$Log_{10} 10=1$ so $10^1=10$

$Log_4 1/16=-2$ so $4^{-2}=1/16$

The expressions $y=a^x$ and $y=log_a x$ are inverses of each other.

*Properties of logarithms*

Altman, S., Comparatore, C and Kurzrok, L. (2008, p.41)

- If **m** and **n** are positive numbers and **a**, a base, with: $\mathbf{m=a^x}$; $\mathbf{n=a^y}$, then the product of these numbers is: $\mathbf{m.n=a^x.a^y=a^{x+y}}$.

The above expression shows that the product **m . n** can also be expressed as a power of the base **a.**

According to the definition of the logarithm: $\log_a(\mathbf{m}\ .\ \mathbf{n}) = x + y$, since $x=\log_a \mathbf{m}$ and $y=\log_a \mathbf{n}$ we conclude that:

$\log_a(\mathbf{m}\ .\ \mathbf{n})=\log_a\mathbf{m}+\log_a\mathbf{n}$

That is, the logarithm in base **a** of the product of two numbers is the sum of the logarithms of those numbers, in the same base.

- Since $\frac{\mathbf{m}}{\mathbf{n}} = \boldsymbol{a}^{x-y}$ we can infer that:

$\log_a\frac{\mathbf{m}}{\mathbf{n}} = \log_a \mathbf{m} - \log_a \mathbf{n}$

The logarithm, in base **a**, of the quotient of two numbers is the difference of the logarithm of the dividend and the logarithm of the divisor, in the same base.

Other properties of logarithms are:

- The logarithm, in base **a**, of the **r**-th power of a number **m** is equal to the exponent **r** multiplied by the logarithm of that number in the same base.

$\log_a(\mathbf{m}^r)=\mathbf{r}.\log_a \mathbf{m}$

- The logarithm, in base **a**, of the **r**-th root of a number **m** is equal to the reciprocal of the index **r** multiplied by the logarithm of that number in the same base.

$\log_a(\sqrt[r]{\mathbf{m}})=\frac{1}{\mathbf{r}}.\log_a \mathbf{m}$

Task "The million"

Using the formula found above for the compound interest amount, we will find out, justifying, at what time (number of periods) we will reach the amount of one million pesos:

$C_n=C\ (1+i)^n$

Replacing the data:

$1000.000=100.000\ (1+0,10)^n$

$1000.000/100.000=1,1^n$

$10=1,1^n$

We are looking for the number to raise 1.1 to give a value of 10

Precisely looking for the exponent is looking for a logarithm, as explained above.

In order to find the value of n we apply logarithm to both members and the property of the logarithm of a power:

$\log 10=\log 1,1^n$

$\log 10=n\ .\ \log 1,1$, clearing n we are left with:

$n=\log 10/\log 1.1$

solving with the calculator gives us:

$n=24,16$

If in the table of values made in activity 1 we add, for example, up to n=30, we would observe that in period 25 the amount already exceeds one million pesos.

| x | y |
|---|---|
| 1 | 110000 |
| 2 | 121000 |
| 3 | 133100 |
| 4 | 146410 |
| 5 | 161051 |

| | |
|---|---|
| 6 | 177156,1 |
| 7 | 194871,71 |
| 8 | 214358,881 |
| 9 | 235794,769 |
| 10 | 259374,246 |
| 11 | 285311,671 |
| 12 | 313842,838 |
| 13 | 345227,121 |
| 14 | 379749,834 |
| 15 | 417724,817 |
| 16 | 459497,299 |
| 17 | 505447,028 |
| 18 | 555991,731 |
| 19 | 611590,904 |
| 20 | 672749,995 |
| 21 | 740024,994 |
| 22 | 814027,494 |
| 23 | 895430,243 |
| 24 | 984973,268 |
| 25 | 1083470,59 |
| 26 | 1191817,65 |
| 27 | 1310999,42 |
| 28 | 1442099,36 |

*Logarithmic Function*

Task "Richter Scale"

An earthquake is measured with an amplitude 392 times greater than A0. What is the magnitude of this earthquake using the Richter scale, in tenths? Graph R=f(A) using Graphmatica software. Justify.

Use the Richter scale equation.

$$R = \log\left(\frac{A}{A_0}\right)$$

A - the measure of the earthquake wave amplitude

A0 - amplitude of the smallest detectable wave (or standard wave)

R -earthquake intensity[1]

Exponential and logarithmic equations

Exercise

Altman, S., Comparatore, C and Kurzrok, L. (2008, p.70 and 71)

Find the values of x that verify each of the following equalities.

a. $9^{2x-3} : 3^{x-2} = 27.81^{1-x}$
b. $4^{2x-1} : 8^{2-x} = 16.2^{2-2x}$
c. $3^{2-x}.(5^{x+1})^{2-x}.6 = 3^{2x}:7$
d. $\log_5(3x-4) = -2$

[1] extracted and prepared by https://www.montereyinstitute.org/courses/DevelopmentalMath/TEXTGROUP-15-19_RESOURCE/U18_L4_T2_text_container_es.html

e. $(4^{1-x})^{2-3x}.2^{x+1}=8^{x}.\frac{1}{2}$
f. $\log_3[(4x-1).(x-1)]=1$
g. $\log_4(2x-3)+\log_4(5-x)=2$
h. $\log_3(x-5)+\log_3(2x+3)=-1$
i. $\log_{\frac{1}{3}} 27^{432}=x$
j. $x=\log_2(\frac{1}{8})^{-387}$

Justify all previous resolutions.

Rtas:

a. x=11/7
b. x=14/9
c. x=1,9178192036x=–2,96563
d. x=-101/75
e. x=1
f. $x=\frac{5+\sqrt{57}}{8}$ó$x=\frac{5-\sqrt{57}}{8}$
g. There is no solution.
h. x=18
i. x=-1296
j. x=1161

*Exercise and problem guide*

Solve the following exercises and problems of the book of the mentioned pages, justifying in all cases. Altman, S., Comparatore, C and Kurzrok, L. (2008, p.73,74,75,76,77,78,79 and 80)

***Annex 2***

DIAGNOSTIC TASKS

1. Solve by applying properties:
   1. $a^{200}:a^{50}=$
   2. $a^{200}.a^{50}=$
   3. $(a^{200})^{50}=$
2. Extract common factor:
   1. 2x+ax=
   2. $-3x^2+2x=$
3. Find the value of h in the following equations:
   1. h/2-3h=4
   2. 2.(h/2-5)=-3(h+1/2)
4. Convert to an equivalent expression without using the fractional exponent:
   1. $m^{3/2}$
   2. $r^{4/5}$
5. Given the following function:

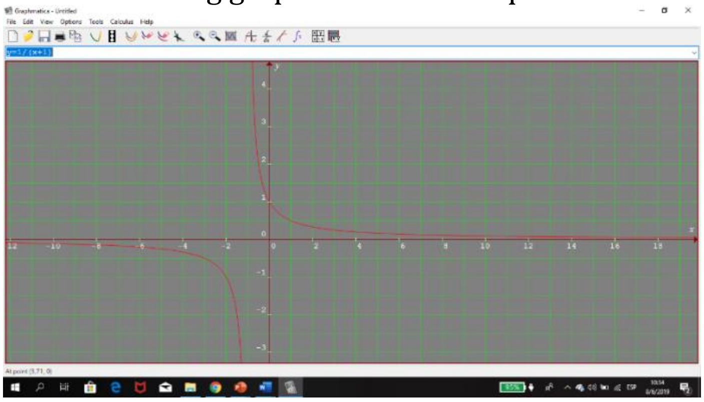
   $f(x)=(x-3)(x+1)^2$

a. Graph it

b. Classify it

c. Indicate its domain, image, zeros, intervals of positivity and negativity, intervals of growth, decrease and constancy.

6. Given the following graph indicate the equations of the asymptotes:



7. Given the function $f(x)=2x-2$ find $f^{-1}(x)$
8. Solve by the method of equalization the following system of equations:
   $2x+3y=4$
   $-4x+y=6$

9. Given the following function: $f(x)=-2x^2+3x+1$, find the coordinates of the zeros, the equation of the axis of symmetry and the coordinates of the vertex.

10. Answer T or F. Justify

a. In the following exercise the commutative property was applied:
   $a.b.=c.d. \Rightarrow a.b.e=c.d.e$ being $e\neq 0$

b. 30/1000=30%

c. The symbol $\Rightarrow$ means the same thing as the symbol $\Leftrightarrow$

d. $(-2)^{-3}= -8$

e. $\sqrt{\sqrt[3]{64}}=2$

f. On the calculator the calculation: $2^8/ (4*8)$ results in 512

g. -2 is a solution of the equation $| x - 2| = 4$

***Annex 3***

DIAGNOSTIC TEST

EXPONENTIAL AND LOGARITHMIC FUNCTION

1. Pose and solve the following problem:

The number of bacteria N(t) of a sample, after a time t in seconds, is obtained with the expression $N(t)=e^{k.t}$. If the population growth rate k is 25 individuals per second, how many bacteria will be in the sample after 10 seconds?

2. Solve the following equation by applying the corresponding properties:

$5^{x+1}+5^x=150$

3. Solve by applying the corresponding properties:

$\log_5 (1/125:625)$

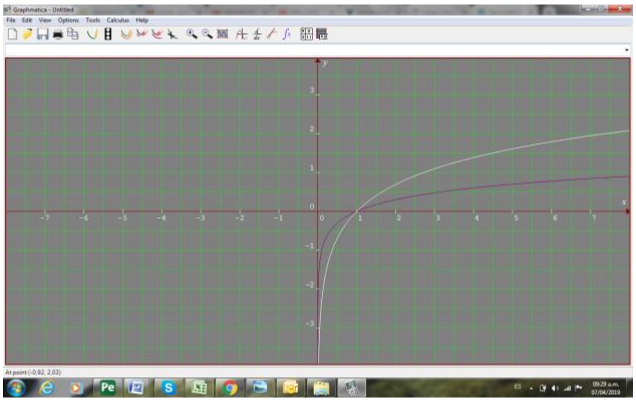
4. Indicate the domain of the following function and represent it graphically:

$f(x)=\log_5(-5x+10)$

5. Solve the following logarithmic equation by applying the corresponding properties:

$\log (3/2 -x)=\log 3/2 - \log x$

6. Observe the figure and identify which graph corresponds to y=ln x and which to y=log x



***Annex 4***

Interviews with students

Initial, exploratory or diagnostic interview

Task to which it applies: "Juggling balls" and "Christmas wreaths."

| Ask | Some student responses |
|---|---|
| How did you solve the tasks for the first time? | Student 1: "Listening and doing what the teacher said." Student 2: "We did the activity among all of us (we chatted about it)." |
| What difficulties did you have in performing the procedure? | Student 1: "More than anything the difficulty was seen because I didn't have the issue well oiled." Student 2: "I did not know the meaning of some words (image, domain, injective, overjective, bijective)." |
| Could you explain to me how area and volume are plotted as a function of radius? | Student 1: "Yes, by replacing in the formula" Student 2: I used the formula $A=r^2.\pi$ and $V=4/3\ \pi.\ r^3$ |
| Could you explain to me how the functions are analyzed? | Student 1: "Yes, knowing the rankings and looking at the chart." Student 2: "They are analyzed by their intervals of positivity and negativity and their growth and decay." |

Development or follow-up interview

Task to which it applies: "Fixed term"

| | |
|---|---|
| Describe the procedure you used to calculate the interest. | Student 1: "We replace the data in the formula $C=C_0 (1+i)^n$" Student 2: "Rule of 3 with the percentage of interest" |
| What mathematical concepts did you need to find it? | Student 1: "We require knowledge of logarithms and exponential equations." Student 2: "Knowing how to clear" |
| How did you recognize if the graph belonged to a function? | Student 1: "Locating the values, if it gives interest the function increases." Student 2: "I don't remember." |
| What were your difficulties in arriving at the formula? | Student 1: "At first I had a hard time relating it to the reasoning, but after a while, I could." Student 2: "None" |

Development or follow-up interview

Task to which it applies: "The Million"

| | |
|---|---|
| Could you summarize the task in your words and explain the concept of logarithm? | Student 1: "It is the way to find an exponent in an equation where it is unknown" Student 2: "The concept is that if you don't mark base it's 10." |
| Why was it necessary to apply the definition of logarithm and the properties of logarithms to solve the task? | Student 1: "because otherwise you would not be able to clear the n which was boosted" Student 2: "To realize that without logarithm we can't go on with the problem." |

Development or follow-up interview

Task to which it applies: "Richter Scale."

| | |
|---|---|
| Have you ever observed an earthquake? | Student 1: “Not in person, but I saw videos on TV.”<br>Student 2: “Yes” |
| Did you immediately understand the statement? | Student 1: “no, the truth is that it cost me a lot”<br>Student 2: “It took me a while to understand it.” |
| What strategy do I use to solve it? | Student 1: “Cancel as much as possible. Cleaning up the equation”<br>Student 2: “I applied the formula that was in the homework” |

Final interview
Self-assessment questions

| | |
|---|---|
| How do the tasks you performed contribute to your mathematical education? | Student 1: “It helps me practice the subject more.”<br>Student 2: “With the knowledge of terms and formulas”. |
| What was the most significant thing you discovered while performing the tasks? | Student 1: “It helps me practice the subject more.”<br>Student 2: “It really reinforces my mathematical knowledge.” |
| How did problem solving enable your understanding of the concept of exponential and logarithmic functions? | Student 1: “It allowed us to deconstruct the subject and help logical thinking.”<br>Student 2: “Many times relating abstract situations such as numbers and accounts to real situations helps to understand them much better.” |