

**COLECTIVE MATHEMATICAL ERRORS OF ELEMENTARY STUDENTS IN
THE BILINGUAL SYSTEM IN HONDURAS
ERRORES MATEMÁTICOS COLECTIVOS EN ESTUDIANTES DEL NIVEL PRIMARIO
DEL SISTEMA BILINGÜE EN HONDURAS**

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ABSTRACT

Keywords:

mathematical errors, collective errors, learning, reflection, educational quality.

Learning Mathematics constitutes one of the most prevalent lines of research in recent times in the field of educational sciences. The importance of this knowledge for life is a secret to no one, justified by its use in multiple social, business, academic tasks, among others. This research arises due to the inadequate knowledge that bilingual students at the primary level have about the errors they can make and those they make while learning mathematics, which lead the purpose of this scientific text: to qualitatively analyze the collective mathematical errors that second grade students make while learning mathematics at an Elementary bilingual school will help improve their learning. A non-experimental study of qualitative and descriptive design is conducted. To collect data, interviews are conducted, questionnaires and academic tests are administered for an intentional sample of 100 second grade students and their corresponding processing to achieve the intended objective. The reflections of the 6 second grade teachers complement the analysis of the errors that students make during the teaching-learning process they facilitate. As a result, it stands out that at a collective level, students make errors in the thinking processes to solve problems that require Algebraic Thinking skills, which, according to the teachers, is largely due to the low level of understanding of English as a second language that constitutes an educational requirement of the school.

RESUMEN

Palabras clave:

errores matemáticos, errores colectivos, aprendizaje, reflexión, calidad educativa.

El aprender Matemática constituye una de las líneas de investigación de mayor prevalencia en los últimos tiempos en el campo de las ciencias educativas. Para nadie constituye un secreto la importancia de este saber para la vida, justificado por su uso en múltiples tareas de carácter social, empresarial, académicas, entre otras. Esta investigación surge, a razón del inapropiado conocimiento que poseen los estudiantes bilingües del nivel primario de los errores que pueden cometer y los que cometen mientras aprenden matemática; lo que trajo como propósito el

desarrollo de este texto científico: analizar cualitativamente los errores colectivos matemáticos que cometen mientras aprenden matemática los estudiantes de segundo grado del nivel básico de un centro educativo bilingüe para la mejora de su aprendizaje. Se realiza un estudio no experimental de diseño cualitativo y descriptivo. Para la recolección de datos se realizan entrevistas, se administran cuestionarios y pruebas académicas para una muestra intencional de 100 estudiantes de segundo grado y su procesamiento correspondiente para dar cumplimiento al objetivo previsto. Las reflexiones de los 6 docentes de segundo grado complementan al análisis de los errores que cometen los estudiantes durante el proceso de enseñanza aprendizaje que ellos gestionan. Como resultado, sobresale que a nivel colectivo los estudiantes cometen errores en los procesos de pensamiento para la resolución de problemas que requieren habilidad de Pensamiento Algebraico; lo cual, según los docentes, se debe en gran medida al nivel bajo de comprensión del inglés como segunda lengua que se instrumenta como exigencia educativa del centro educativo.

Introduction

Teaching and learning mathematics have been a line of research that has proliferated over time; there have been multiple attempts by teachers, managers and families to make the study of this science a problem that is already a problem in the diversity of contexts and nations. However, the teaching and consequently the learning related to this science is still far below international standards, which defines where academic efforts should be directed by all those involved in the education of children, adolescents and young people.

According to Rojas (2020), the educational system in Honduras has been no exception, as it has faced significant historical and structural obstacles, resulting in a deficit in the development of essential skills for the country's progress. The inadequate provision of efficient educational services presents a major obstacle for the Honduran public education system in terms of its main product for society: students who only possess outdated knowledge of the subjects tested.

The findings of the standardized tests in Mathematics and Spanish (reading) of students in the first, second and third cycles of basic education on a global scale highlight a remarkable situation within the Honduran educational system. This challenge refers to the main outcome that the system intends to provide to society, i.e., students who have inadequate learning levels and possess only a minimal understanding of the tested subjects. The current state of stagnation requires the implementation of several initiatives aimed at improving children's academic performance within the national primary education framework (Rojas, 2020).

In the process of searching for solutions we found that: "Research conducted in recent years has shown the importance of focusing attention not only on students' correct answers, but also on the errors they make" (Socas, 2007, p. 20). Additionally, there are consistent error patterns at two levels: at the individual level, where people exhibit a high degree of regularity in their approach to solving similar exercises and problems, and at the collective level, where individuals make similar errors in certain phases of their learning (Rico, 1995).

On the other hand, "We speak of error when the student performs a practice (action, argumentation, etc.) that is not valid from the point of view of the school mathematical institution" (Godino et al., 2003, p. 69). Consequently, identifying and analyzing the collective errors of bilingual students in order to correct these difficulties, which form the scaffolding of knowledge for learning mathematics, is a good starting point for improving the quality of education.

According to Socas (1997), students' errors due to the complexity of mathematics encompass the acquisition of concepts and application of mathematical procedures. The "knowing" and "doing" of mathematics go hand in hand in the process. The present difficulty becomes a major obstacle to learning and, if not identified early during the primary level, can become entrenched over time. In this regard, Hernández-Suárez et al. (2017) state that many students who have trouble learning basic concepts and managing them find it difficult to advance in their education.

Errors are inherent to the mathematical learning process of students. Errors are empirical information that we constantly encounter in the teaching and acquisition of mathematics; they form an enduring component of these processes. Since the main objective of mathematics teaching in the educational system is to ensure that all students acquire a thorough understanding of the subject, it is clear that any incorrect answers or solutions to the questions posed are considered an indicator of significant deficiencies and shortcomings in the achievement of this objective. For this reason, the analysis of

mathematical errors is an important topic in Mathematics Education, since its historical trajectory has been marked by diverse approaches and interests (Kilpatrick et al., 1998).

In addition, according to Kilpatrick et al. (1998), with the increase in popularity of mathematics classes in recent years, people in our country are once again interested in studying and investigating the mistakes children make in school. "However, since recent times, there has been considerable progress in mathematics education research, and there is a growing interest in achieving a clear scheme of interpretation and prediction of errors and misconceptions" (Kilpatrick et al., 1998, p. 83).

"Studying and analyzing errors made by students has recently emerged as a major line of study and research in Mathematics Education, with considerable implications for much of the fields of study in our area" (Kilpatrick et al., 1998, p. 85). For the author, there are recent studies and researches related to errors in mathematics learning that mention studies dedicated to the curricular treatment of errors including works dedicated to the didactic organization of mathematics teaching that include errors as a relevant data. Another line of study is related to teacher training and the observation, analysis, interpretation and management of student errors.

Socas (2011) cites his own work (Socas, 1997), to remind us of the different difficulties that students may face in learning mathematics the different difficulties that students may face during mathematics learning, which she mentions in five categories: a) Complexity of Mathematics; b) Thinking processes; c) Teaching processes; d) Students' cognitive development processes; e) Affective and emotional attitudes towards mathematics.

Difficulties due to the Complexity of Mathematics

According to Socas (1997), the complexity of mathematics encompasses the acquisition of concepts and application of mathematical procedures. The "knowing" and "doing" of mathematics go hand in hand in the process. The present difficulty becomes a major obstacle to learning and, if not identified early during the primary level, can become entrenched over time. In this regard, Hernández-Suárez et al. (2017) state that many students have trouble learning basic concepts and managing them. This makes it difficult for them to advance in their education.

A clear example is presented by Juarez and Lopez (2016) who concluded in their study that many students entering college carry bad foundation in Algebra, since they did some exercises and did not realize that they were fractions, so they did not follow the elementary school method of adding and subtracting fractions. Elementary school children beginning to learn fractions make the mistake of adding numerators and denominators. The same mistake was made by these college students.

On the other hand, Araya et al. (2018) state that abstraction and generalization in mathematics is a likely source of learning difficulties. This assertion is made in relation to students entering college and the challenges associated with mathematical content. By analyzing the mathematical material, it is possible to foresee the degree of potential difficulty it may present and to determine the factors that must be taken into account to facilitate its teaching. Sometimes the error is not due to a lack of information, but to the fact that the student uses knowledge that in some cases is valid, but in others is misapplied. This happens when the student uses knowledge that is legitimate in some circumstances, but not in others where it is misapplied.

Difficulties in Thought Processes

Difficulties in Thinking Processes refer when moving from one knowledge to another, either from one subject to another or one branch of Mathematics to another. In

this regard, Caballero and Juárez (2016) confirmed in their study analysis of algebraic errors among first-year students at a public institution in Puebla, Mexico that when students are presented with a variety of algebraic problems, they often rely on the arithmetic knowledge they have acquired in the past. This can make it difficult for them to adapt to the change from arithmetic to algebra, which in turn can cause them difficulties.

In practice, it has been observed that these difficulties are more evident when students move from one educational level to another. As a result of the teachers' reflections, they attribute it to the fact that, on occasion, in the previous course the students did not learn certain concepts that are necessary to move from one topic to another. These learning gaps create gaps that must be closed with mini-lessons and reviews of previous topics that serve as a basis for knowledge of new ones (Caballero & Juárez, 2016).

On the other hand, when content is taught superficially and with little opportunity to put learning into practice, it creates a problem for creating the connections necessary for learning. In this regard, Guizado et al. (2022) concludes that the development of mathematical thinking is crucial both for progress in the sciences and in everyday life; we frequently count, estimate, create, analyze, question, and guess; we always think about something or do it out of curiosity.

Mathematical thinking is a complex process; its development requires knowledge about the pillars that compose it: the first is related to numerical thinking of arithmetic processes. The second pillar is geometric thinking, which is characterized by processes related to the ability of movement, location and shape. The third pillar is metric thinking, which refers to scaled measurements to measuring instruments. The fourth pillar is random thinking, which includes the ability to manage data and probabilities, and the fifth pillar is variational thinking, which works with algebraic content related to the ability of equivalence, order and regularity (Guizado et al., 2022).

In a modern sense, we must recognize that mathematical thinking encompasses thinking about mathematical topics and advanced thought processes in a variety of contexts (abstraction, justification, visualization, estimation, and reasoning under hypothesis). This thinking, therefore, must operate on a complex network of advanced and basic concepts and procedures. These concepts are underpinned by practices (Cantoral et al., 2015).

Lozada and Fuentes (2018) in their study examining how problem-solving techniques can enhance the growth of mathematical reasoning and suggesting strategies for incorporating them into the classroom tells us that recently, there has been general agreement on the need to teach Mathematics to improve cognitive skills, beyond simply imparting mathematical concepts. The focus is shifting from developing problem-solving skills to improving critical thinking in problem solving. Many authors have suggested problem-solving methods, but there are limited concrete proposals to help teachers use these methods and heuristic resources to effectively implement problem-solving strategies that promote improved mathematical thinking.

Difficulties in the Processes of Teaching Mathematics

The cognitive development processes of students involved in learning are to know and analyze difficulties, obstacles and errors in learning Mathematics. And among the teaching aspects are to analyze and design ways and situations of teaching mathematical contents and to foresee their consequences. Finally, the evaluation of learning, which includes analyzing and designing situations to determine and assess the mastery of learning (Socas, 2011).

Following this idea, the didactic aspect equips teachers to design, elaborate, develop and evaluate the different curricular programs in order to be able to analyze, situate and sequence each of the content blocks. Teachers must also know and know how to use the resources available to them to adapt them to the learning needs of their students. In this sense, Barallobres (2016) highlights the importance of mathematical didactics and considers the contextual and institutional dimensions of mathematics learning as fundamental for the explanation of the learning phenomena of this discipline, thus giving meaning issues a central place; consequently, the identification of mathematical errors made by students will help the achievement of objectives associated with the mathematical education of students.

According to Pochulu (2009), after a careful examination and evaluation of the errors observed in students' work, he concludes that an important part of these errors stem from the teaching and learning processes of Mathematics, characterized by the following factors:

- Excessive reliance on algorithmic techniques or routines lacking theoretical foundations
- Use of insignificant rules as prerequisites for performing arithmetic calculations or solving equations
- Emphasis on algebraic concepts at the expense of practical problem-solving skills
- Presentation of disconnected and poorly integrated content, particularly in relation to other subjects
- Insufficient emphasis placed on cultivating skills related to critical interpretation of data and graph analysis
- Excessive reliance on visual representations that hinder the creation of conceptual understanding
- Excessive focus on numerical approaches to problem solving

Difficulties due to Students' Cognitive Developmental Processes

Cognitive processes are a factor that influences student competence in mastering mathematical concepts and skills. According to González-Nieves et al. (2016), studying Mathematics is a complicated way of learning that needs to be supported by timely neurobiological maturation. This neurobiological maturation should allow the student to reach a certain degree of cognitive development, which in turn facilitates mathematical learning. In that sense, it is imperative to take into account what is age-appropriate to teach. That said, learning expectations must go hand in hand with the intellectual capacity of students, especially in learning activities that require the use of critical thinking which at an early age is very limited.

According to Ariza et al. (2021), acquiring competence in mathematical problem solving is a fundamental and unbiased component of primary education. Problem solving has been considered part of students' higher cognitive ability, which has limited its use as a specialized subject for teaching and learning in research assessing how well mathematics is taught. The comprehension of arithmetic problems in primary education is influenced by the cognitive processes of primary school students, the textual characteristics of arithmetic problems and the specific levels of cognitive performance in text comprehension.

According to Ariza et al. (2021), the initial stage of development occurs between 6 and 7 years of age, specifically during the 1st and 2nd years of primary school. The first level of cognitive processing involves the ability to detect and understand local information such as places, characters and activities. It also includes the ability to replace

a term with a synonym, identify key words, search for supplied material expressly related to a topic, select data for a problem without superfluous data; understand the given and the claim(s) in given problems.

Second level (applicative): reformulating expressions; associating a problem with examples based on specific elements; paraphrasing the circumstances presented in the problem; choosing relevant facts in simple problems; forming transitive associations; condensing data using hyperonyms; determining the objective and repercussions of activities; inferring traits; evaluating attitudes.

The third phase of creativity involves abstracting the links between a topic and its illustration, creating graphical models for given problems, deriving meaning from inferences about parts and wholes, and coordinating equalities to solve difficulties. solving basic problems using graphs or counting; creating simple problems using equalities, illustrations, and visual diagrams.

Students' Affective Attitudes Toward Mathematics Learning

Affective and emotional attitudes toward mathematics are a common problem in students. The important thing to know is that the teacher has a great deal of influence on this issue. What is most interesting is that even students at older ages do not overcome this emotional challenge to mathematics. The good news is that students who can recognize their own mistakes can also demonstrate better academic performance (Juarez & Lopez, 2016).

It can be inferred that self-regulated learning is a strategy of great impact on students' academic performance. Self-regulated learning has been found to have a beneficial impact on student achievement at various educational levels, from elementary school through college. There is a positive correlation between a student's level of self-regulated learning and his or her academic performance. Conversely, there is a negative correlation between a student's low level of self-regulation of learning and his or her academic performance. Aside from academic performance, research has shown that self-regulated learning can improve students' motivation and self-confidence, leading to greater engagement in the learning process and better academic outcomes. The above findings are consistent with previous research on the impact of self-regulated learning tactics on school performance. Thus, to achieve superior academic performance, it is imperative to prioritize the implementation of self-regulatory learning strategies ((Fauzi & Widjajanti, 2018).

According to Pochulu (2009), teachers claim that students frequently read a sentence that is almost always incomplete and demands an immediate response. If they can't get the information in a few seconds, they immediately approach the teacher or a classmate who knows how to solve the problem. Gómez (1995) explains that this student's attitude is natural, since the teacher solves an exercise and presents the "clean" solution, without indicating the "draft" process by which the solution was reached. Consequently, the student believes that he must also find the solution "clearly" and is unaware that, in order to solve an exercise, he must have an adequate method or strategy, so he looks for shortcuts. These shortcuts divert you from the correct path and cause you to make mistakes.

On the other hand, Godino et al. (2003) states about the challenges in terms of student motivation, it is possible that, despite the fact that the class preparation procedure and the activities that have been offered are sufficient, the students may not be able to cope with them due to lack of motivation. This may be due to problems with each student's self-esteem or their own academic background. In relation to the affective response to mathematical errors, how teachers affectively project students' mathematical

errors influences how those students will feel when they make errors in the future (Barquero, 2023).

According to De la Osa (2016), students' attitudes and values are shaped by mathematics because they ensure solidity in its foundations, security in its procedures, and confidence in its results. All of this instills in children a conscious and favorable disposition to undertake actions that lead to the resolution of the daily challenges they face.

Therefore, for De Nicolás et al. (2016) in their study on the difficulties of students in teaching, recommend that it is important to carry out teaching, learning and evaluation processes that focus on the acquisition of skills and knowledge and try to improve affective and attitudinal factors. This is because the teacher's main job in the classroom is to help students improve their mathematical reasoning, their ability to form and solve problems, communicate their mathematical ideas, and see how the different parts of mathematics fit together.

Based on the results of one study, one can see the importance of the affective factor, which means that the student must be interested in mathematics, and all participants agreed that the role of the teacher is very important for this to happen (Barquero, 2023). To the above, it is important to add that students' lack of knowledge of primary mathematical information detracts from their self-confidence and is aggravated by their attitude towards arithmetic and their nervousness when solving problems (Nortes & Nortes, 2017).

Method

Study Design and Sample

The research design is non-experimental, qualitative and descriptive. The participants of this study comprised a non-probabilistic sample and a purposive sample of 100 students at the second-grade level at the elementary level with students between the ages of 8-9 years and 6 mathematics teachers. The institutional documents that evidenced the diagnosis of the educational center in the area of Mathematics and its relationship with regard to the management of students' collective errors were analyzed. Other instruments used were questionnaires that were previously reviewed by a group of experts for their required evaluation and approval. The questionnaires were used to interview teachers about their experience with the handling of mathematical errors for the analysis of these errors in the areas of numeration, statistics and algebraic thinking. The teachers' questionnaires are applied in Google forms format to collect qualitative information on the perceptions of students' collective mathematical errors and to analyze through the interviews, the handling of these errors by teachers and students. Student questionnaires are administered on paper for ease of completion. Statistical calculations and data analysis of the student and teacher surveys were carried out using the *Statistical Package for Social Sciences (SPSS)*, in the latest version 26. Finally, with the written academic tests, the students' collective errors were identified according to those that reflected the highest frequency. They were then analyzed according to the criteria of Socas (2011): Complexity of Mathematics, Thinking processes, Teaching processes, Cognitive development processes of students, Affective and emotional attitudes towards mathematics. Each academic test question validated by the iReady Math program (Curriculum Associates, 2020), corresponds to a standard from the Common Core U.S. Common Core standards in the following areas of mathematics: Numeration, Algebraic Operations and Statistics.

Research Objective and Hypothesis

The objective of the study was to qualitatively analyze the collective errors made by bilingual students during the learning of mathematics in a private bilingual school in Honduras, which guarantees, among its most innovative aspects, the improvement of educational quality. The following hypothesis was put forward as the hypothesis of the present study: Knowing the collective mathematical errors that students make during learning helps to improve their educational process during their stay at the private bilingual center in Honduras.

The independent variable was: The students' collective mathematical errors, which was analyzed in the collective mathematical errors identified in second grade students using the five categories proposed by Socas (2011) on the different difficulties that students may face during mathematics learning, which he mentions in five categories: a) Complexity of Mathematics; b) Thinking processes; c) Teaching processes; d) Cognitive development processes of students; e) Affective and emotional attitudes towards mathematics.

Study Participants

The sample for this study was simple purposive at the second grade primary level with the participation of 6 teachers and 100 students (6 teachers and 20 students from each of the 5 second grade sections of the Sampedrana International School).

The iReady Math program test was applied, which included a total of 12 problems and equations from the areas mentioned above. The sample was a group of 100 students in the second grade of primary education at the Sampedrana International School, of which 45 are girls and 55 are boys, and all of them were included in this study. The age range of second grade students is between 8 and 9 years old.

The second grade sections were chosen because the teachers have a highly qualified professional profile and have worked for the institution for more than 5 years. Also, they have actively participated in the changes in the improvement of Mathematics and work in an organized way and as a team they have expressed their desire to find real solutions to the current situation regarding the academic performance of their students.

Data Collection Instruments Used

Questionnaires, interviews and the academic test were used as research instruments for this research work. Questionnaires were drafted for teachers and students and included closed multiple-choice questions and some open-ended questions where they could express their reflections and were analyzed by the researcher. The questions were designed according to the 5 categories of Socas (2011) and were sent through a Google forms link that could be completed individually by each teacher participant. The questions were analyzed and approved by a group of experts and then administered and analyzed. The purpose of the questionnaires was to collect teachers' and students' perceptions of the errors they make in learning mathematics.

The researcher personally interviewed a purposive sample of at least 36 students who made errors on the academic test to obtain qualitative information about the collective errors students made on the Mathematics test. The 6 second grade mathematics teachers who teach the mathematics class were also interviewed. A purposive sample of students who made the collective errors on the academic test was selected to collect qualitative information to analyze the errors.

A second interview was conducted with the 6 teachers about their reflections on the students' mistakes in terms of the aspects that attracted their attention about the results of the academic test and, on the other hand, the questions that arose as a result of

the mistakes made by the students during the learning of Mathematics. Finally, what strategies do they use for handling students' mathematical errors by: Subject complexity, thought processes, cognitive development, teaching processes and affective attitudes of students (Socas, 2011).

The instrument called the i-Ready Math program diagnostic test validated by Curriculum Associates (2020) for second grade students was selected. The test consisted basically of a series of mathematical problems in English covering the first unit of the content block for Mathematics of the first bimester of second grade. The problems were organized by standard and the multiple-choice response required the student to reflect on the procedure he or she followed to arrive at the answer. The test was administered in English to 100 second graders at the primary level in groups of 20 students per teacher.

In the results of the diagnostic tests, the errors and successes per student were recorded for each question. Subsequently, according to the frequency of errors in each question, collective errors were identified and tabulated in bar graphs. Once the collective errors were identified, with the interviews of the students who made the errors, they were analyzed and classified according to the categories of Socas (2011).

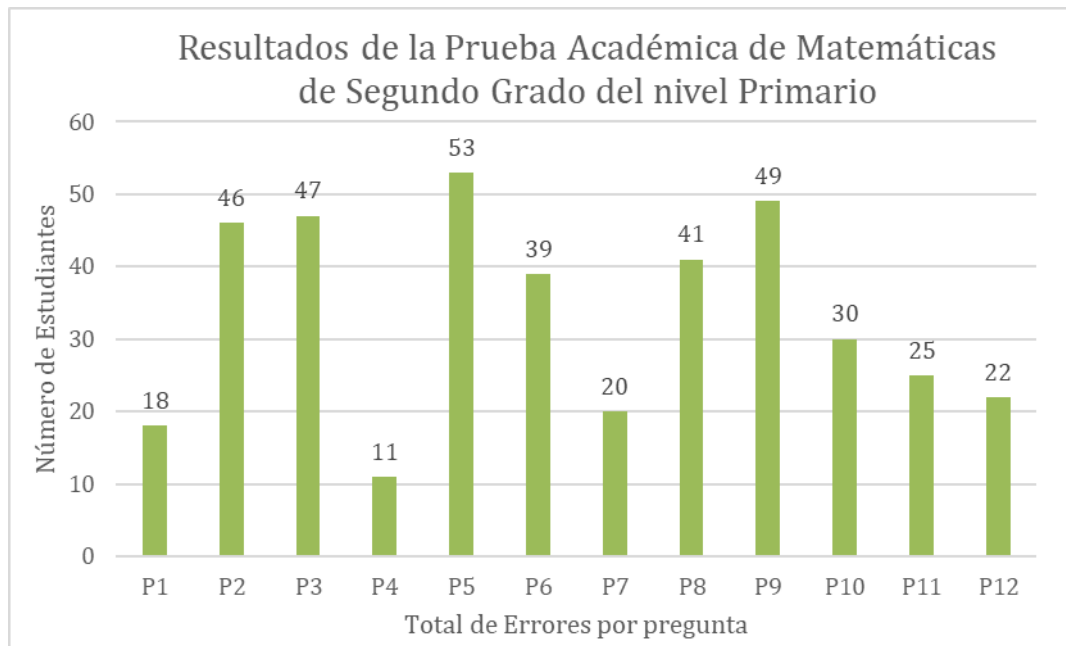
The data obtained after the application of the instruments were processed, organized, coded and statistically tabulated. The answers of the academic test were analyzed by question identified with the highest frequency of error, this was done through the elaboration of tables and graphs using the Excel sheets program and allowed us to answer the research questions. Statistical calculations and analysis of the teacher and student surveys were performed using the program using the *Statistical Package for Social Sciences (SPSS)*, in the latest version 26 at the time of the analysis of this research.

Results

The results of the academic tests showed that students presented mathematical errors in problems that required solving problems by interpreting bar graphs (P2), solving problems by applying the operation of addition or subtraction (P3), solving problems with mixed two-step operations (P5) and sums with double numbers (P9).

Figure 1

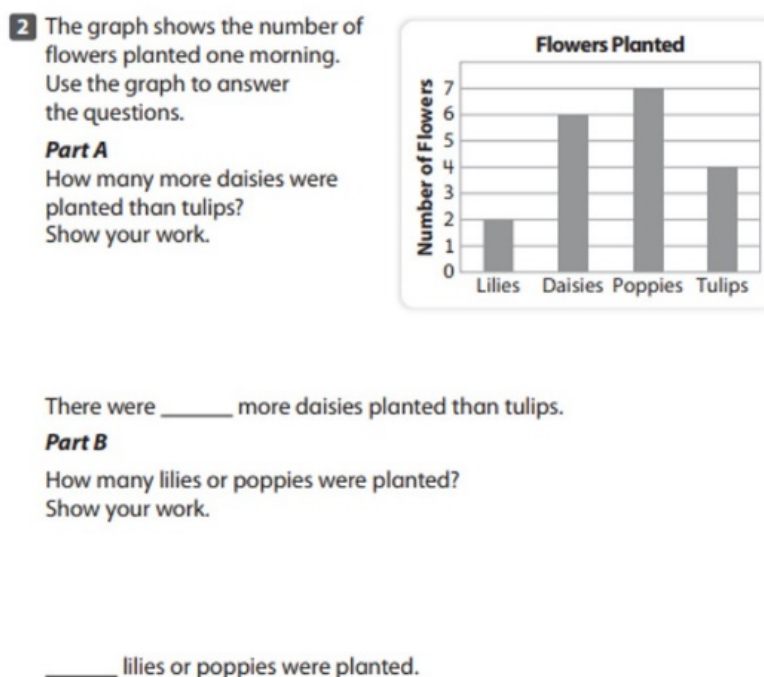
Mathematical Errors of second grade bilingual students according to academic test



In solving question number 2 (Q2) on problems with bar graphs and corresponding to the statistics standard (2.MD.D.10) of the Common Core standards, teachers read the problem aloud for students, then they read it independently to solve the problem related to the number of flowers planted in a garden. The problem included a bar graph representing the number and type of flowers that were planted in the garden. Among them were: Daisies and Tulips. To solve it correctly, students had to understand what the problem posed to them. After analyzing it, understanding what a bar graph meant, knowing how to read the bar graph and solving the problem using the information on the bar graph.

Figure 2

Question about Troubleshooting bar graphs (P2)



Note. Source: Curriculum Associates (2020)

In summary, for the resolution of the statistics problem (2.MD.D.10) of the Common Core standards, the difficulties encountered by the students were the following: a) They could not read the bar graph to know how many flowers there were of each species b) Those who managed to read the graph were not sure what to do with the information c) They did not know if they should add or subtract d) The students used the wrong data to perform the operation. Faced with this difficulty, some left the problem incomplete and failed to get to the step of verifying their answer.

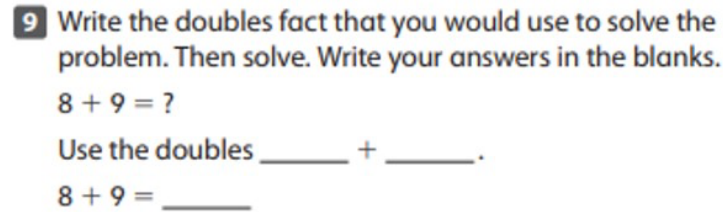
In solving problem 3 (P3) of the Operations and Algebraic Thinking standard (2.OA.B.2) of the Common Core standards, teachers read the problem aloud for students, then they read it independently to solve the problem related to an everyday life problem it stated: "Sean has 10 books. He has 6 more books than Kali. how many books does Kali have?" Teachers read aloud the problem, students listened attentively and understood it. Students read the problem again and independently to understand it. They then used the correct operation using the numbers they had to build the equation with to correctly subtract $10 - 6 = 4$ and verify their answer. When interviewing the students who made mistakes in solving the problem, they acknowledged that: a) They could not understand the problem due to difficulties in reading and understanding English b) They stated that they were not sure of the operation to be used c) The others accepted that they had not checked their answers and made a mistake when performing the calculation.

On the other hand, collective errors were found in the resolution of question 5 (Q5) on 2-step mixed operations problems corresponding to the Operations and Algebraic Thinking standard (2.OA.A.1) of the Common Core standards that stated: "Jan places 14 markers on a table. 8 markers fall off the table. Riese puts 5 of those markers back on the table. how many markers are on the table now?". In this problem, students made the following mistakes: a) They wrote the equation, but incompletely, reflecting only one

operation instead of two. b) They wrote the complete equation, but applied the wrong operations. c) They also acknowledged not having checked their answers.

Figure 3

Question on double number addition strategy (Q9)



Note. Source: Curriculum Associates (2020)

Finally, in solving problem 9 (P9) of the Operations standard (2.OA.B.2) of the Common Core standards regarding sums with double numbers, students had to write double numbers as a strategy to easily add two large numbers to solve the problem. When interviewing the students who made the mistake, they stated that: a) They did not know the meaning of double numbers b) They did not understand the purpose of the problem with double numbers.

Table 1

Collective mathematical errors of second grade bilingual students

Problem	Errors found
Solving Problems with bar graphs and corresponding to the Statistics Standard (2.MD.D.10)	a) They could not read the bar graph to know how many flowers there were of each species b) Those who were able to read the graph were not sure what to do with the information c) They did not know whether to add or subtract d) The students used incorrect data to perform the operation.
Problem Solving for Operations and Algebraic Thinking standard (2.OA.B.2)	a) They could not understand the problem due to difficulties in reading and understanding English b) They stated that they were not sure of the operation to be used c) The others accepted that they had not checked their answers and made a mistake when performing the calculation.
Solving 2-step mixed operations problems corresponding to the Operations and Algebraic Thinking standard (2.OA.A.1)	a) They wrote the equation, but incompletely, reflecting only one operation instead of two. b) They wrote the complete equation, but applied the incorrect operations. c) They also acknowledged not having checked their answers.
Solving equations with double numbers corresponding to standard Operations (2.OA.B.2)	a) They did not know the meaning of double numbers b) They did not understand the purpose of the problem with double numbers.

As a result of the qualitative analysis of the students' collective errors according to the results of the academic test, the analysis of the types of collective

errors committed by the students follows. According to the above graph, it can be concluded that the four types of errors that were identified were: a) They used an incomplete equation b) They did not understand the concept of doubles or factor families c) They added or subtracted wrongly (precision error) and d) They applied the wrong operation to solve the problem (addition instead of subtraction or vice versa).

During the interviews with students in which errors due to incomplete equations were analyzed, they stated that: "The problem was very long, I thought we only had to solve with 2 numbers which is what we have always done in class". On the other hand, others simply did not realize that there was a third number in the problem that had to be included in the equation and did not pay attention to the whole problem.

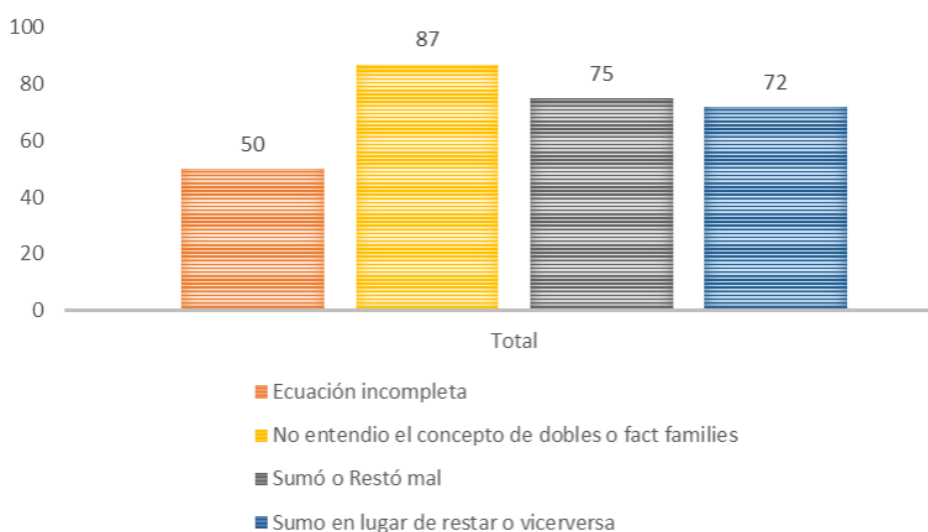
On the other hand, students who made mistakes on the question about factor families and double numbers admitted not remembering what the stated concepts meant, but admitted that the topic had been discussed in class. They expressed it as follows: "I don't quite remember what we learned because it was a long time ago, but I know we saw it in class."

Students who added or subtracted wrongly made calculation errors and admitted that: a) They did not check their answers b) They miscounted the drawings they used to solve the problem c) They solved mentally to finish the problem on time d) They were confident they could solve it mentally.

Finally, those who applied the wrong operation (added instead of subtracting or vice versa) acknowledged that: a) They did not understand the problem because of difficulty in understanding English b) Others confused the term "more" and thought they had to add. It should be explained that, in English, the word "more" means "more". However, the question: "How many more?" is used to ask "How many more?" and means that they should subtract.

Figure 4

Analysis of Collective Mathematical Errors according to the types of errors of second grade students of a Bilingual Educational Center



On the other hand, according to the teachers' perceptions, the four types of collective mathematical errors that students generally made were due to a difficulty in understanding the problem, which they attributed to a low level of understanding of English as a second language. The teachers' conclusion was derived from the fact that the problems were read in English and aloud by the teachers, and could be repeated for the students to better grasp the information. In conclusion, these errors were manifested as follows: 50% of the students left the equation incomplete, 87% did not understand the concept of doubles or factor families, 75% added or subtracted wrong (precision error) and 72% applied the wrong operation to solve the problem (addition instead of subtraction or vice versa). According to the analysis the teachers concluded that: "Transferring knowledge to new contexts is complicated for many students, especially when learning such a complex subject in a second language."

In this regard, Graus and Perez (2017) cited Radatz (1979) to refer to a classification of 5 categories of errors from information processing, among which he mentions errors due to language difficulties. This includes understanding concepts, symbols and vocabulary specific to mathematics that compares to learning a second language.

Finally, precision mathematical errors were the least frequent for second grade bilingual students at the school. They were able to solve the algorithms using visual strategies such as the number line, use of concrete or manipulatives and drawings.

Discussion and Conclusions

With the results found on the qualitative analysis of the students' collective errors we confirm our following hypothesis *H1= Knowing the collective mathematical errors that students make during learning helps to improve their educational process, during their stay in the private bilingual center in Honduras.*

We conclude that collective errors affect the learning of Mathematics of second grade bilingual students of an international educational center in different ways; being the most influential factor, according to the academic tests and with the complement of the teachers' opinions, the students' thinking processes due to the difficulty of transferring learning to new contexts.

In this regard, in the interviews with teachers, no major emphasis was identified on the variety of strategies to reinforce students' thinking processes. With the above, it is intended to achieve a teaching focused on lasting learning that is transferable and that empowers the student to identify and correct his or her mistakes independently.

We also concluded that knowing the collective mathematical errors that students make during learning helps to improve their educational process in a private bilingual school in Honduras, because the thought processes are a factor that influences the student's competence in mastering mathematical concepts and skills that are needed to learn the next grade or the next level.

According to the students surveyed, most students recognize that their teachers always know their mistakes, help them overcome them, explain well when they do not understand and teach them about the mistakes they can make. In this regard, it is indisputable that the teaching-learning process of Mathematics should promote student autonomy in learning, offering techniques to connect previous knowledge with new

concepts, facilitating the acquisition of new knowledge through personal experience and encouraging the development of mathematical reasoning (Lozada & Fuentes, 2018).

On the other hand, the resolution of mathematical problems reveals collective errors that require the ability of Algebraic Thinking. On this topic, Montero and Mahecha (2020) cite Blanco and Caballero (2015) who present a more contemporary viewpoint by introducing an integrated model of problem solving that encompasses affective and cognitive components. This model is organized in five distinct phases. Each stage of the process has a distinct purpose: cultivating understanding and control over one's cognitive reactions; identifying potential approaches that result in a resolution; implementing previously chosen strategies; evaluating responses; and ultimately, contemplating the task accomplished. It is important to note that this progression remains focused on the overall goal of enabling students to develop their own problem-solving methodology.

In relation to the students' handling of their mistakes and according to the surveys on students' perceptions, more than 50% of them perceive that they always know how to correct their mistakes. However, of these, only 19% admit that they never have trouble correcting them. The above confirms the problem of how second grade students handle mathematical errors, since although more than half of the students perceive that they always know how to correct their errors, very few admit that it is never difficult for them to correct them. That said, we need to create action plans in terms of teaching strategies that equip students with the knowledge and skills for identifying and dealing with error more independently to eradicate the limitation in the difficulty they perceive in correcting their errors.

As a result of the qualitative analysis of the students' collective errors with the academic test administered, the analysis of the types of errors made identified in the students as a result of the processes they followed for the resolution of these errors. In conclusion, the four types of errors that were identified both in the academic tests and their subsequent analysis with from the interviews conducted with the students who made the errors are:

- The incomplete equation
- They did not understand the concept of doubles or families of factors
- They applied the wrong operation to solve the problem (sum in instead of subtraction or vice versa)
- Wrong addition or subtraction (accuracy error)

Additionally and taking as a reference the criteria of Socas (2011) on the categories of mathematical errors of students and according to the perceptions of second grade bilingual students recognized the difficulty in understanding the new topic and remembering what they know to learn a new topic. These exercises require the transfer of learning from knowledge to practice for new contexts. Likewise, for teachers and according to their perceptions, they also identified it as a collective mathematical error of second grade bilingual students.

On the other hand, when content is taught superficially and with little opportunity to put learning into practice, it creates a problem for creating the connections necessary for learning. According to Rico (2008), when engaging in student-oriented learning tasks, it is crucial to establish the necessary conditions for analyzing the task requirements. This involves taking guiding actions, such as assessing existing knowledge, identifying gaps in knowledge, determining available data, understanding the task requirements, and designing a solution strategy. Reflection, as a component of cognitive processes, allows the student to develop an understanding of the methods used in an activity, as well as the strategies and results obtained.

For Pochulu (2009), we must also recognize that many of the mistakes that students make in mathematics are not due specifically to the topic being developed, but to a lack of prior knowledge that is transferred to the new content being addressed. In this regard, second grade teachers agreed that they were unaware of the level of depth with which second grade students were taught and reinforced in those topics when they were in first grade.

In relation to thinking processes, according to the results of the students' surveys, 67% of them perceived that sometimes they find it difficult to understand new words and 55% find it difficult to understand figures and symbols in Mathematics. To assist students in the use of academic language, language routines that promote understanding of the problem are suggested. Among those routines, repetitive reading is very effective in getting students to understand the problem by reading it 3 times, being intentional each time they read it. For example: The first reading your approach is to answer the question: What is the problem about, the second reading your approach is to answer the question: What are we trying to find? And the third question: what are the important quantities and ratios? (Curriculum Associates, 2020).

Curriculum Associates (2020) recommends differentiated instructional strategies called tiered language for students learning mathematics in English. The strategies are described as follows: Read aloud and in chorus the problem being posed, use drawings to represent the concepts being taught, work with a partner to solve the problem, show the steps they followed to solve it, and explain to share aloud how they solved it.

Another collective mathematical error that bilingual students make while learning mathematics was found in the thought processes for solving mathematical problems.

Multiple scholars have recognized the crucial role of problem solving in the educational process. The potential of the learning situation is underutilized, with a strong focus on learners acquiring patterns to improve their problem-solving skills, neglecting its importance for the development of thinking, particularly mathematical cognition (Lozada & Fuentes, 2018).

For purposes of continuity of the research process, it is recommended that the same study be carried out in the lower and higher grades to obtain a stepwise idea of the evolutionary behavior of the error by level. This is due to the fact that the students of the school learn English as a second language and their level of English improves every year in terms of comprehension and oral and written communication of the foreign language.

On the other hand, it is recommended to study the types of errors per student according to their classification according to the Response to Intervention Model (RTI). With the description of each student's profile and the type of mathematical errors that are identified, the way is opened for a more personalized pedagogical attention and the grouping of students by abilities to facilitate differentiation.

Some limitations of the present study were the lack of recent studies on the subject of mathematical errors in elementary school students, and in particular, in students who learn mathematics in a second language. On the other hand, the use of mathematical language during the interviews, data analysis and discussion of the results with the participants. As a result of the conversations and in order to understand the ideas of others, it was necessary to clarify the concepts that were being used in order to match them with the ideas of others. For the purpose of similar studies, it is recommended to prepare a list of mathematical vocabulary to be used by the analysis and discussion group. In this way, we ensure that everyone speaks the same language and that ideas flow easily.

Another limitation on the qualitative analysis of the students' errors consisted in the fact that it was difficult for the students who made the errors to explain the procedure they followed and how they handled the error. This is due to the fact that, according to

their developmental maturity, some have better communication skills than others. Therefore, in order to collect the necessary information, more students had to be interviewed than were planned for the qualitative analysis of the collective errors of second grade students. In specific cases, teachers supported students through dialogue.

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